

# 1 Delta Function Potential

A particle of mass  $m$  is in a region of potential  $V(x) = -V_0L\delta(x)$ . First, use the variational principle to get an upper bound on the particle's ground state energy. Then, solve for the ground state energy exactly and show that it is less than the determined bound.

# 2 Gravitational Perturbations

Consider a particle of mass  $m$  is in a one-dimensional harmonic potential  $V(x) = \frac{1}{2}m\omega^2x^2$ . Suppose an gravitational field is applied, so that the potential is shifted by an amount  $V'(x) = -mgx$ . Find the lowest non-vanishing correction to the energy levels. Also, solve the system exactly and show that the resulting energies are consistent with the perturbation results.

# 3 Compton Scattering

A photon of energy  $E_\gamma$  is incident on a particle of mass  $m$ , at rest in the lab frame. Find  $E_{\gamma'}$ , the energy of the photon after it has scattered as a function of the scattering angle. Also, find the maximum energy that a back-scattered photon can have. (This maximum will be independent of  $E_\gamma$ . If you're clever, you can do this in 5 lines or less.)

